Precalculus B Study Guide
Credit by Exam for Credit Recovery or Acceleration

The exam you are interested in taking is designed to test your proficiency in the relevant subject matter. You should be thoroughly familiar with the subject matter before you attempt to take the exam. This CBE CR/A Study Guide can help you prepare for the exam by giving you an idea of what you need to review. You can check your familiarity level by reviewing the Texas Essential Knowledge and Skills (TEKS) for this course. (See below.) To refine your skills, you can refer to any of the state-adopted textbooks.

Texas Essential Knowledge and Skills (TEKS)

Every question that appears on this exam is derived from the knowledge and skills statements and student expectations within the Texas-mandated standards, the Texas Essential Knowledge and Skills (TEKS). You can view the TEKS for this exam online via the following link: http://ritter.tea.state.tx.us/rules/tac/chapter111/ch111c.html#111.42 Refer to section (c), Knowledge and skills, 1A–5N. Throughout this guide, you'll see TEKS references. These refer to the numbers listed under (c) Knowledge and skills; for example, 1A or 3B.

Materials Needed

You will need to bring a graphing calculator. The formula sheet at the end of this study guide will be provided during the exam. You can also bring this formula sheet with you to your exam. If you are taking a print exam, you must bring a #2 pencil to complete the exam. You will receive a computer-graded answer sheet when you arrive at the testing center. The proctor will provide scratch paper.

Exam Structure

You will be allowed **3 hours** to complete this exam. The Precalculus B exam consists of 50 multiple-choice questions worth a total of 100 points. The exam covers a wide variety of topics. To help you study, we have isolated 4 key topics and provided study tips and sample questions for each. You can expect several multiple-choice questions on each of the following topics:

Topic 1: Plan and Solve Real-world Problems
Topic 2: Parametric Equations and Conic Sections
Topic 3: Trigonometry and Vectors
Topic 4: Sequences and Series

Formula Chart

You will be provided with the formula chart at the end of this study guide. Familiarize yourself with these formulas and how to use them to solve problems based on the TEKS above.

Scholastic Honesty

When you arrive at the testing center you will be asked to carefully read the exam rules and sign a statement agreeing to take the exam in accordance with the rules. This is called the Examinee's Certification. The following is a copy of these rules:

<div align="center">

Examinee's Certification
</div>

This certification must be signed *before* the exam is administered and then returned with the completed examination attached, or credit for the exam will not be given.

Scholastic dishonesty is a serious academic violation that will not be tolerated. Scholastic dishonesty encompasses, but is not limited to:

- copying from another student's work;
- using an unauthorized testing proctor or taking the exam at an unauthorized testing location;
- using materials not authorized by a testing proctor;
- possessing materials that are not authorized by a testing proctor, such as lessons, books, or notes;
- knowingly using or soliciting, in whole or Topic, the contents of an unadministered test;
- collaborating with or seeking aid from another student without authorization during the test;
- substituting for another person, or permitting another person to substitute for oneself, in taking a course test or completing any course-related assignment;
- using, buying, stealing, or transporting some or all of the contents of an unadministered test, test rubric, homework answer, or computer program.

Evidence of scholastic dishonesty will result in a grade of *F* on the examination and an *F* in the course (if applicable).

At the testing center, you will be asked to sign a statement that says
you have read the above and agree to complete the examination with scholastic honesty.

General Study Tips

The bulleted lists and sample questions in this study guide can assist you in preparing for the exam. It is a fairly complete guide for studying, but does not cover every item on the test. Ultimately, you should use the TEKS to guide your exam preparation.

Additional Study Tips

The following information provides direction for your studies. For each part, you will find study tips and sample questions to give you a general idea of the types of questions you can expect to see on the exam.

Topic 1: Plan and Solve Real-world Problems

This is a broad topic that can relate to any type of question. Students should show the ability to create a plan for problem-solving, explain their reasoning, and solve problems arising in everyday life.

Study Tips for Topic 1:
This topic relates to TEKS 1A–1G. Familiarize yourself with those TEKS, and then be prepared to demonstrate knowledge of the following topics:
- Solve word problems about the workplace and every day interactions in society
- Justify your solution to a problem
- Explain if a solution is or is not reasonable
- Create and use tables, graphs, number lines, Venn Diagrams and other representations to organize information

Sample Questions for Topic 1:
The following are sample questions. You can find the correct answers listed after the questions, but try answering the questions without looking at the answers first to check your comprehension.

DIRECTIONS: Select the BEST responses to the following questions.

1. The wind speed s (in miles per hour) near the center of a tornado is related to the distance d (in miles) the tornado travels. The speed can be found using a logarithmic model such that the speed is 65 more than 93 times the common log of the distance it has travelled. If a tornado with a wind speed of 200 miles per hour struck your city, how far did the tornado travel? Round your answer to the nearest mile.

   A. 15 miles
   B. 28 miles
   C. 141 miles
   D. 279 miles

Topic 2: Parametric Equations and Conic Sections

This topic relates to your understanding parametric equations, polar coordinates, conic sections, as well as properties and graphs of each.

Study Tips for Topic 2:

This topic relates to TEKS 3A–3I. Familiarize yourself with those TEKS, as well as the 2nd and 3rd pages of the formula chart, and then be prepared to demonstrate knowledge of the following topics:

- Graph a set of parametric equations given as x (t) and y (t)
- Convert between parametric equations and rectangular relations (such as a data table, equation, graph, or mapping diagram)
- Use parametric equations to model and solve real world problems
- Graph points using polar coordinates
- Graph polar equations by plotting points and with your graphing calculator
- Recognize each of the possible shapes formed when a plane intersects a double-napped cone
- Recognize and identify the connection between the locus definition of conic sections and the corresponding equation (e.g. a circle can be considered as all the points that satisfy a given equation and at the same time a circle can also be considered as the set of all points that are a specific distance away from a given center point)
- Understand how to use the formulas for equations of circles, ellipses, hyperbolas, and parabolas to graph conic sections and to identify various properties of that conic section
- Use the characteristics of an ellipse, such as its foci, to write an equation
- Use characteristics of a hyperbola, such as its center, to write an equation

Sample Questions for Topic 2:

The following are sample questions. You can find the correct answers listed after the questions, but try answering the questions without looking at the answers first to check your comprehension.

DIRECTIONS: Select the BEST responses to the following questions.

2. Choose the graph that correctly locates the polar coordinates $\left(-3, \dfrac{4\pi}{3}\right)$.

A.

B.

C.

D.
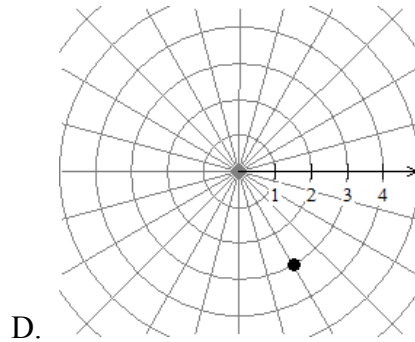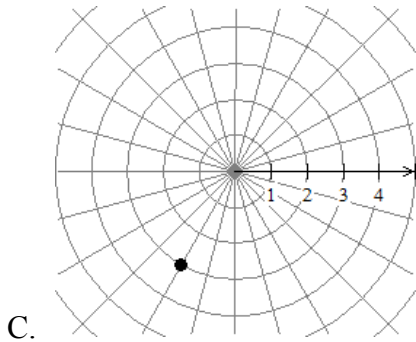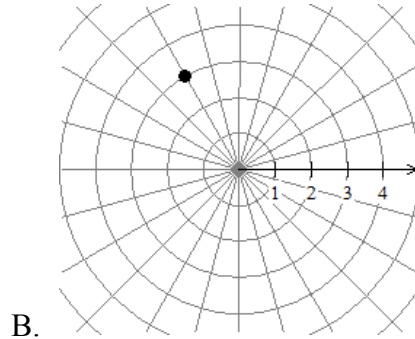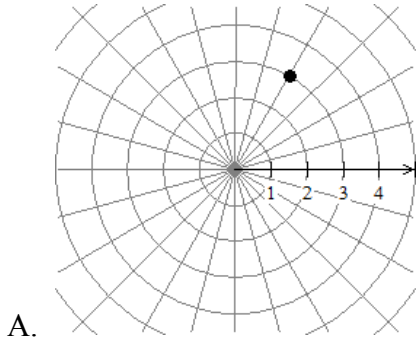
3. Which of the following polar equations will produce a circle that has a diameter of 12 and goes through the origin?

       I. $r = 12$
       II. $r = 12\cos\theta$
       III. $r = 12\sin\theta$

A. I only
B. III only
C. II and III
D. I, II, and III

4. Which set of parametric equations describes a line segment that starts at (9, 5) and ends at (3, 15)?

A. $\begin{cases} x = 9 - 3t \\ y = 5 + 5t \end{cases}$

B. $\begin{cases} x = 9 + 5t \\ y = 3 + 15t \end{cases}$

C. $\begin{cases} x = 5 + 5t \\ y = 9 - 3t \end{cases}$

D. $\begin{cases} x = 3 + 6t \\ y = 15 + 10t \end{cases}$

Topic 3: Trigonometry And Vectors

This topic relates to your understanding of the use of vectors and trigonometry to solve problems involving real world situations and triangles that do not have a right angle.

Study Tips for Topic 3:
This topic relates to TEKS 4F–4K. Familiarize yourself with those TEKS, and then be prepared to demonstrate knowledge of the following topics:
- Reference the formulas for the trig ratios as well as the Law of Sines and Law of Cosines and know how they are used with triangles in problem solving
- Represent angles in terms of their directional bearing and compare that angle's representation in standard position to its representation in directional bearing (e.g. 065° or N 22° W)
- Use trigonometry to solve real-world problems using directional bearing
- Use the Law of Sines to solve real-world problems
- Use the Law of Cosines to solve real-world problems
- Use vectors to model situations involving magnitude and direction
- Recognize the conventions and meanings of drawing and labeling vectors (for example the meaning of drawing vectors head-to-tail versus drawing them tail to tail)
- Recognize and use symbols and diagrams to represent addition and multiplication of vectors
- Apply vector addition and multiplication to solve real-world problems

- When labeling sides and vertices in triangles, capital letters are used as vertices for angles, and the same letter written in lowercase is used to represent the length of the side opposite from that angle as shown in the diagram below. Use these naming conventions when applying formulas and drawing triangles to solve word problems
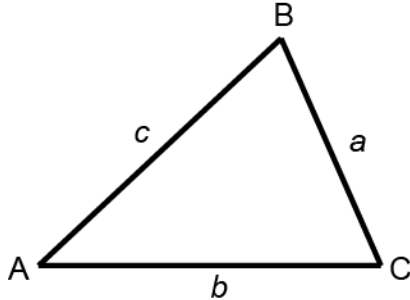
Sample Questions for Topic 3:
The following are sample questions. You can find the correct answers listed after the questions, but try answering the questions without looking at the answers first to check your comprehension.

DIRECTIONS: Select the BEST responses to the following questions.

5. A jet leaves an airport at 1:00 pm with a course of 140° and a speed of 530 mph. At 3:00 pm, the jet pilot changes the course to 220° and maintains the same speed until 6:00pm. Determine how far the jet is from the airport at 6:00 pm.

   A. 278.12 miles
   B. 812.01 miles
   C. 1750.22 miles
   D. 2058.41 miles

6. For five days, several hikers walk the distance and direction described by the vector $\langle 5, -2 \rangle$ each day. The next day, they walk the distance and direction described by the vector $\langle -1, 8 \rangle$. At the end of the six-day trip, what is their bearing from their starting location?

   A. S85°E
   B. S5°E
   C. S8°E
   D. S82°E

7. Which vector is NOT parallel to $\begin{pmatrix} 7 \\ 17 \end{pmatrix}$?

   A. $\begin{pmatrix} 0.7 \\ 0.17 \end{pmatrix}$

   B. $\begin{pmatrix} -7 \\ -17 \end{pmatrix}$

   C. $\begin{pmatrix} 3.5 \\ 8.5 \end{pmatrix}$

   D. $\begin{pmatrix} 21 \\ 51 \end{pmatrix}$

Topic 4: Sequences and Series

This topic relates to your understanding of using various types of sequences in series to solve problems, both with and without real world context. You are expected to understand how to apply the formulas for sequences and series provided on the formula chart, as well as the vocabulary and notation used to describe terms and the relationships between terms.

Study Tips for Topic 4:
This topic relates to TEKS 5A–5F, 5M, and 5N. Familiarize yourself with those TEKS, and then be prepared to demonstrate knowledge of the following topics:
- Evaluate finite arithmetic sums and finite geometric series when written in sigma notation
- Understand the differences between arithmetic and geometric sequences
- Represent arithmetic and geometric sequences using recursive notation
- Calculate the nth term and n^{th} partial sum of an arithmetic series
- Represent sums of series using sigma notation
- Calculate the nth term of a geometric series, the nth partial sum of a geometric series, and the sum of an infinite geometric series when it exists
- Apply the Binomial Theorem for the expansion of a power of a given binomial expression to solve problems
- Use the binomial theorem or Pascal's Triangle to identify the coefficients of an expansion of a binomial to a given power
- Familiarize yourself with the trigonometric identities given on the formula chart and know how to use them to simplify larger and more complicated trigonometric expressions
- Generate trigonometric equations to solve real-world problems
- Use trigonometric identities to simply expressions and solve equations
- Identify all the solutions to a trigonometric equation on a given interval (such as $0 \leq \theta \leq 2\pi$) using inverse trig functions, the unit circle, and/or graphing calculators
- Use periodicity to write the general form of all the real solutions to a trigonometric equations

Sample Questions for Topic 4:
The following are sample questions. You can find the correct answers listed after the questions, but try answering the questions without looking at the answers first to check your comprehension.

DIRECTIONS: Select the BEST responses to the following questions.

8. A ball is dropped from a height of 16 feet. Each time it bounces, it returns to a height that is 75% the height from which it last fell. What is the total distance the ball travels?

    A. 64 feet
    B. 112 feet
    C. 128 feet
    D. Infinitely many feet

9. Which of the following expressions are equal to 30,600?

$$\text{I.} \qquad \sum_{n=1}^{45} 30n - 10$$

$$\text{II.} \qquad \sum_{n=5}^{49} 20 + 30(n-5)$$

$$\text{III.} \qquad \sum_{n=2}^{46} -10 + 30(n-2)$$

    A. I only
    B. I and II
    C. II and III
    D. I, II, and III

10. After applying trigonometric identities, which of these is equivalent to the expression below?

$$\frac{\cot u}{\csc u}$$

    A. $\sin u$
    B. $\csc u$
    C. $\cos u$
    D. $\sec u$

Answer Key

Item Number	Correct Answer	TEKS expectation
1	B	1A, 1B, 5H
2	A	3D
3	C	3E
4	A	3C
5	D	4F, 4H
6	A	4F, 4K
7	A	4K
8	B	5E
9	B	5D, 5A
10	C	5M

Trigonometric Functions

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad \tan^2 u + 1 = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$$

Cofunction Formulas

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \tan\left(\frac{\pi}{2} - u\right) = \cot u$$

$$\cot\left(\frac{\pi}{2} - u\right) = \tan u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

Even/Odd Formulas

$$\cos(-u) = \cos u \qquad \sin(-u) = -\sin u \qquad \tan(-u) = -\tan u$$

$$\sec(-u) = \sec u \qquad \csc(-u) = -\csc u \qquad \cot(-u) = -\cot u$$

Triangles

Law of Sines

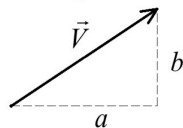$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

Vectors

On this assessment, the following notations are used for vectors.

$$\vec{V} = \langle a, b \rangle = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} = \begin{pmatrix} a \\ b \end{pmatrix} \qquad \text{and} \quad \left\| \vec{V} \right\| = \sqrt{a^2 + b^2}$$

Polar Equations

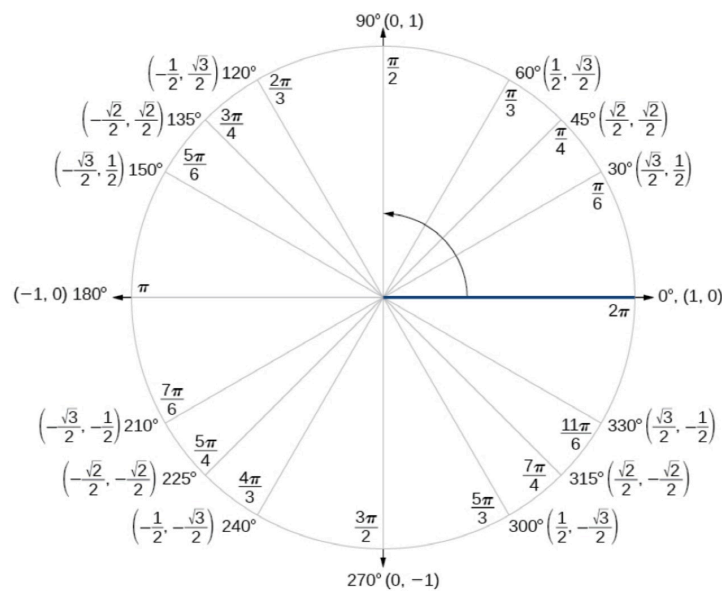| *Polar Coordinates*	*Rectangular coordinates*
(r, θ)	(x, y)
$r = \sqrt{a^2 + b^2}$	$x = r \cos \theta$
$\theta = \arctan\left(\dfrac{b}{a}\right)$	$y = r \sin \theta$

Circles   *Centered at (h, k)*

$$(x - h)^2 + (y - k)^2 = r^2$$

Unit Circle

The content of this image is from OpenStax College, Precalculus.OpenStax College. 23 October 2014. Download for free at http://cnx.org/content/col11667/latest/.

Ellipses   If $a > b$; then $c^2 = a^2 - b^2$ and c is the distance from the center to a focus;

Horizontal Major Axis

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Vertical Major Axis

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$$

Hyperbolas   If c is the distance from the center to a focus, then $c^2 = a^2 + b^2$;

Horizontal Hyperbola

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Vertical Hyperbola

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

Parabolas   If a is the distance from the vertex to the focus;

$$(x-h)^2 = 4a(y-k) \qquad \text{or} \qquad (y-k)^2 = 4a(x-h)$$

Binomial Theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} \qquad \text{or} \qquad (a+b)^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} a^k b^{n-k}$$

Sums

Arithmetic Partial Sum   *Geometric Partial Sum*   *Geometric Infinite Sum*

$$S_n = \frac{n(a_1 + a_n)}{2} \qquad S_n = \frac{a_1 - a_1 \cdot r^n}{1-r} \qquad S = \frac{a_1}{1-r} \text{; where } -1 < r < 1$$